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CFD-Based Correlation Development for Air Side Performance of Wavy Fin Tube Heat Exchangers using 2mm-5mm Tube Diameters

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ABSTRACT

This paper presents new CFD-based correlations for wavy-finned tube heat exchangers with tube diameter ranging from 2mm to 5mm. Herringbone and smooth wavy fin profiles are analyzed in this study. All results are based on Computational Fluid Dynamics (CFD) simulations. The CFD models are verified using the Grid Convergence Index (GCI) method. The methodology implemented in this work consists of analyzing airside heat transfer and pressure drop characteristics by using Parallel Parameterized CFD (PPCFD) allowing one to simulate a large number of fin designs with significantly reduced engineering time. The resulting data is reduced into correlations using a linear stepwise regression algorithm that can be easily implemented in various heat exchanger analyses tools. The new equations for Herringbone fins predicts 96% and 94% of the data for heat transfer and friction within 15% relative absolute deviation, respectively. Similarly, the equations developed smooth wavy fins predict 94% and 93% of the data within 15% relative absolute deviation for heat transfer and friction, respectively.

1. INTRODUCTION

Applications such as heat pumps in cold climates face an extra challenge of frosting in the outdoor unit, which ultimately compromises the overall system performance. For these systems, the commonly known high performance fins like slits and louvers are generally not suitable. It has been shown empirically that louver fins have a poorer performance compared to plain wavy and flat fins, respectively under frosting/defrosting operating modes (Silva et al., 2011; Huang et al., 2014).

Wavy fins are an elegant way of balancing the thermal-hydraulic performance with, and without, frost accumulation, compared to louver and flat fins, which are best suited for each of the operating conditions. Wavy fins are well understood and discussed extensively in the literature (Kays, 1960; Kays & London, 1984; Wang et al., 1999). Although, limited improvements can be done with wavy fins without adding anything to the surface. One way to improve the performance and increase compactness is by reducing the tube size. To the author's knowledge there are practically no studies on airside performance for wavy fins and tube diameters below 5.0mm. Bacellar et al. (Bacellar et al., 2015) proposed a set of CFD-based correlations for smooth wavy fins using tubes with diameters below 5.0mm with relatively low accuracy. The majority of correlations presented in the literature are valid for larger tube diameters as shown in Table 1. In this paper, we present a correlation development framework that leverages automated CFD simulations to investigate a large design space for tube banks in staggered arrangement, for both Herringbone and smooth-wavy fins using tube diameters from 2.0 to 5.0mm.

2. METHODOLOGY

The methodology comprises of leveraging CFD to evaluate the airside dimensionless thermal-hydraulic factors (j and f). The framework (Figure 1) has four main steps including problem specification, modeling, correlation development and experimental validation. There are three sets of automated CFD simulation runs for the Uncertainty Analysis (UA), Design of Experiment (DoE) and Random designs, respectively.

Table 1: Relevant wavy fin correlations in the literature.

Author	Fin type	Tube arrangement	Correlations	Application range	Accuracy (staggered only)
(Webb, 1990)	Flat and wavy	Staggered	Heat Transfer	9.5mm≤Dc≤12.7mm 2≤N≤6	Nu : 10%(96% of data)
(Kim, Yun, & Webb, 1997)	Wavy Herringbone	In-line / Staggered	Heat Transfer / friction	9.5mm≤Dc≤12.7mm 1≤N≤8	j : 10%(92% of data) f: 15% (91% of data)
(Wang, Fu, & Chang, 1997)	Wavy Herringbone	In-line / Staggered	Heat Transfer / friction	Dc=10.3mm 1≤N≤4	j : 10%(94% of data) f: 15% (95% of data)
(Wang, Jang, & Chiou, 1999)	Wavy Herringbone	Staggered	Heat Transfer / friction	7.7mm≤Dc≤16.4mm 1≤N≤6	j : 15%(95.1% of data) f: 15% (97.3% of data)
(Wang, Hwang, & Lin, Empirical correlations for heat transfer and flow friction characteristics of herringbone wavy fin-and-tube heat exchangers, 2002)	Wavy Herringbone	Staggered	Heat Transfer / friction	8.6mm≤Dc≤10.4mm 1≤N≤6	j : 15%(91% of data) f: 15% (85% of data)
(Bacellar, Aute, & Radermacher, 2015)	Wavy Smooth	Staggered	Heat Transfer / friction	2.0mm≤Dc≤5.0mm 2≤N≤10	j : 20%(64% of data) f: 20% (66% of data)

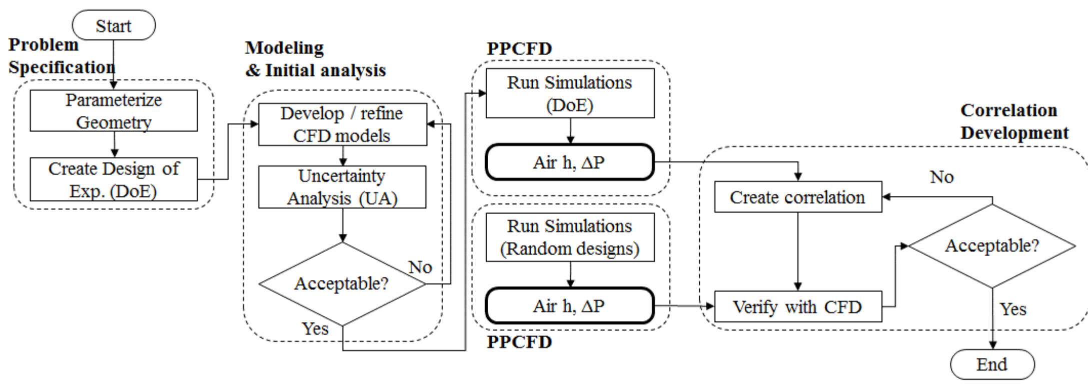
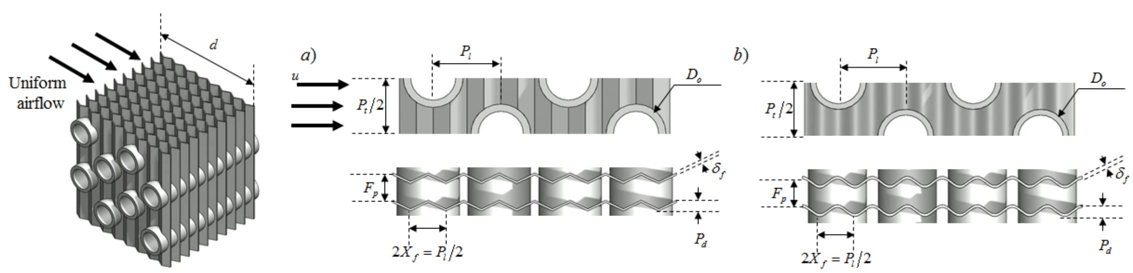


Figure 1: Correlation development framework.

3. PROBLEM SPECIFICATION

This paper investigates the airside thermal-hydraulic characteristics of a fin-and-tube HX in cross flow configuration using Herringbone and smooth (sinusoidal) wavy fins. The design space explored is shown below in Figure 2.



D_o (mm)	P_l/D_o (-)	P_t/D_o (-)	F_p (mm)	P_d/X_f (-)	δ_f (mm)	N (-)	u (m/s)
2.0 - 5.0	1.25 – 4.0	1.25 – 4.0	0.5 – 2.5	0.088 – 0.84	0.05 – 0.1	2 - 20	0.5 – 7.0

Figure 2: Design space: a) Herringbone wave; b) Smooth wave.

4. CFD MODELING AND SIMULATION

The CFD computational domain is a two dimensional cross section segment of the HX, assuming any end effects to be negligible. The inlet boundary has uniform velocity and uniform temperature (300K), whereas the outlet boundary is at constant atmospheric pressure. The upper and lower boundaries are periodic, and the tube walls are at constant temperature of 340K, whilst the fin walls are coupled to the tubes. The faces parallel to the fins on the sides are periodic. The fluid properties use ideal gas model, and the turbulence is evaluated using the k-ε relizable model. The convergence criteria used is 10^{-5}. The near wall region mesh is a fine map scheme with growing layers at a ratio of 1.2 (**Error! Reference source not found.**). The core of the computational domain is a pave mesh scheme with an average element size equal to the last row of the boundary layer mesh.

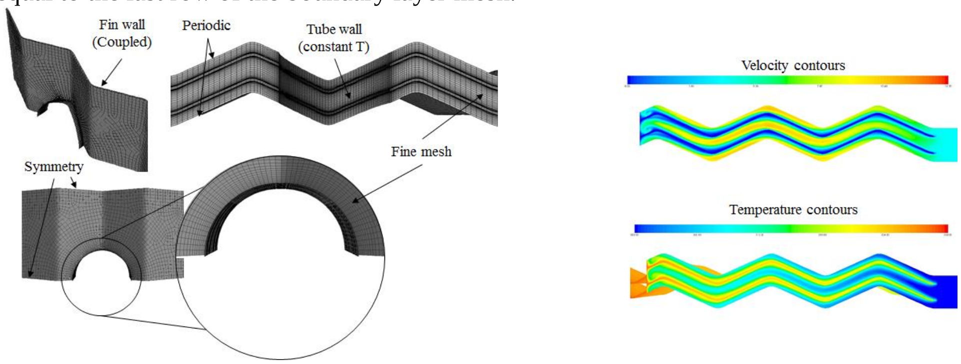


Figure 3: Computational domain, mesh and contour plots.

4.1 Data reduction

Since the CFD models serve to determine the airside thermal and hydraulic resistances, there is no need to account for additional thermal resistances. Thus with constant wall temperature, the capacitance ratio yields $C_{min} / C_{max} = 0$, then the heat transfer coefficient can be easily calculated through ε-NTU method as per equations (1-3). The pressure drop is determined as the difference between inlet and outlet static pressures, assuming that local losses are negligible.

$$NTU = -\ln(1-\varepsilon) = -\ln\left[1-(T_{out}-T_{in})/(T_{wall}-T_{in})\right] \quad (1)$$

$$\eta_o h = UA/A_o = NTU \cdot C_{\min}/A_o \quad (2)$$

$$\eta_o = 1-\frac{A_{fin}}{A_o}(1-\eta), \eta = f(h) \quad (3)$$

The fin effectiveness is obtained using the Schmidt (Schmidt, 1949) approximation method.

4.2 Parallel Parameterized CFD

In order to handle a large number of designs the Parallel Parameterized CFD (PPCFD) (Abdelaziz et al., 2010) is a suitable method that allows one to automate CFD simulations. The code consists of reading and writing data and it communicates with the CFD modeling and simulation environments in an automated fashion. The analyses in this study used the ANSYS® platform, more specifically Gambit 2.4.6 for geometry and meshing, and Fluent 14.5 for the simulation runs.

4.3 CFD Uncertainty Analysis

A common way of determining the CFD model uncertainty is the Grid Convergence Index (GCI) method (ASME, 2009), which quantifies the uncertainty associated to the grid resolution for at least three mesh sizes. Typically, for this type of study the uncertainty analysis is performed on the designs at the boundary of the space since they represent the extreme combinations between the variables. However, this would be an unaffordable additional computational cost. Therefore, we picked a design at the boundary that has high aspect ratios in terms of height, width, and depth and wave amplitude. Such design results in high aspect ratios at the grid element level as well, thus it is expected to have amongst the higher uncertainties. We performed a full Grid Independence Study and GCI analysis on this particular design for air velocity of 2.0m/s (Figure 4).

5. CORRELATION DEVELOPMENT

Here we propose equations (equation 4) based on a multiple regression correlating 7 parameters (equation 6) to the Nusselt number and friction factor (C_f) (equation 5). All values (parameters and response) are on logarithmic form (equations 5 and 6). The equations have a maximum order of 3 and are solved using a stepwise linear regression algorithm. The resulting matrix M contains the power value of each parameter (columns of M) for each of the regression term (rows of M), and the array c contains the coefficients of each term. The matrices M and arrays c are presented in the Appendix at the end of this manuscript. There are two sets of equations for each correlation: one for 2 to 10 tube banks and one for 11 to 20 tube banks. The source data for each fin type consisted of a Design of Experiments with 1300 designs sampled using Latin Hypercube Sampling method.

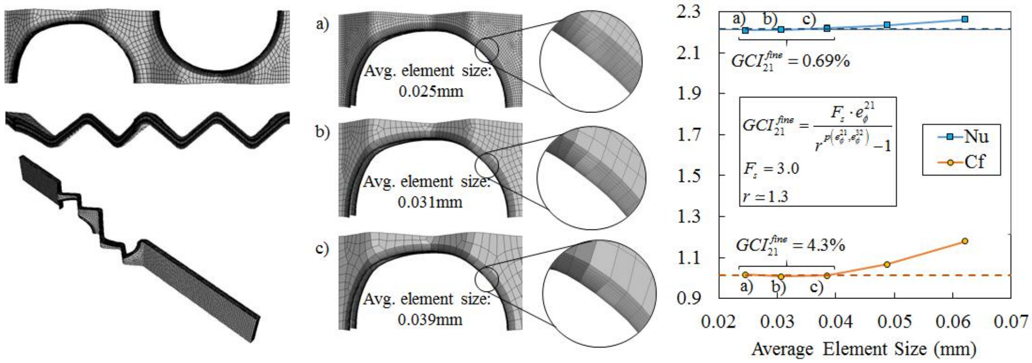


Figure 4: Grid Independence Case of Study.

$$\phi(\underset{\sim}{x}) = \sum_{i=1}^{n_{Rows}} c_i \cdot \left(\prod_{j=1}^{n_{Columns}} x_j^{M_{i,j}} \right), M_{n_{Rows} X n_{Columns}} : \text{Parameters power matrix, } \underset{\sim}{c}_{n_{Rows}} : \text{ Coefficients array} \quad (4)$$

$$\phi_h = \ln\left(Nu_{Dh}\right), \phi_{\Delta P} = \ln\left(C_f\right) \quad (5)$$

$$\underset{\sim}{x} = \left[\ln\left(F_p/D_o\right), \ln\left(P_l/D_o\right), \ln\left(P_t/D_o\right), \ln\left(P_d/X_f\right), \ln\left(\delta_f/F_p\right), \ln\left(N\right), \ln\left(\mathrm{Re}_{D_o,u_c}\right) \right] \quad (6)$$

$$Nu_{Dh} = hD_h/k \quad (7)$$

$$C_f = \Delta P / \left(0.5\rho u_c^2\right) \cdot D_h / \left(4d\right) \quad (8)$$

$$D_h = 4A_c d / A_o \quad (9)$$

$$\mathrm{Re}_{D_o,u_c} = \rho u_c D_o / \mu \quad (10)$$

The non-dimensional heat transfer and pressure drop are calculated for dry air properties at 300K and 1atm. The property values used are presented in Table 2.

Table 2: Air properties.

T (K)	P (kPa)	ρ (kg/m³)	μ (Pa.s)	k (W/m.K)	c_p (J/kg.K)
300	101.325	1.177	1.86E-05	2.57E-02	1005

5.1 Correlation verification

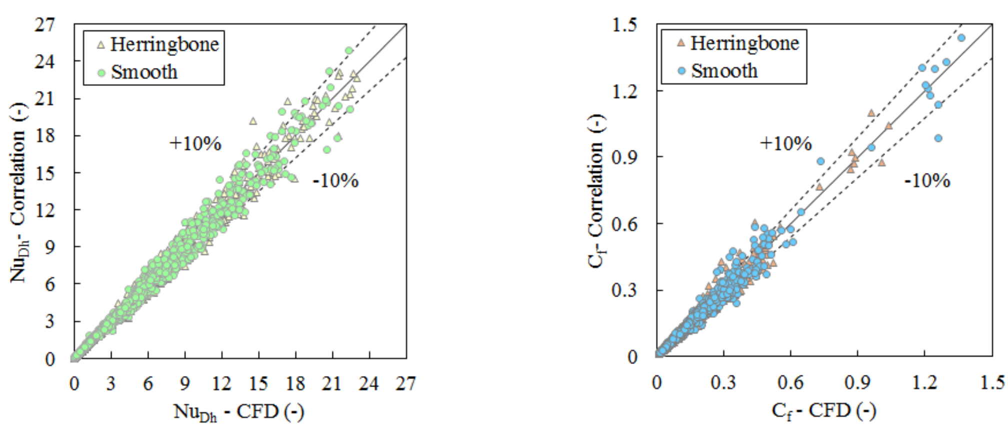


Figure 5: Correlation verification against source data.

Table 3: Correlations fitness.

Fin Type	**Herringbone**		**Smooth**	
Metric	**Nu_{Dh}**	**C_f**	**Nu_{Dh}**	**C_f**
Predicted data (e=10.0%)	84.73%	83.92%	80.84%	81.88%
Predicted data (e=12.5%)	92.63%	91.22%	88.95%	89.43%
Predicted data (e=15.0%)	96.17%	94.54%	94.28%	93.48%
Predicted data (e=20.0%)	99.04%	98.08%	98.97%	97.62%
Predicted data (e=25.0%)	99.56%	99.04%	99.60%	98.97%
R^2	0.9937	0.9881	0.9927	0.9889
Mean Absolute Relative Error	5.344%	5.665%	5.747%	6.038%
Median Absolute Relative Error	3.940%	4.288%	4.253%	4.750%

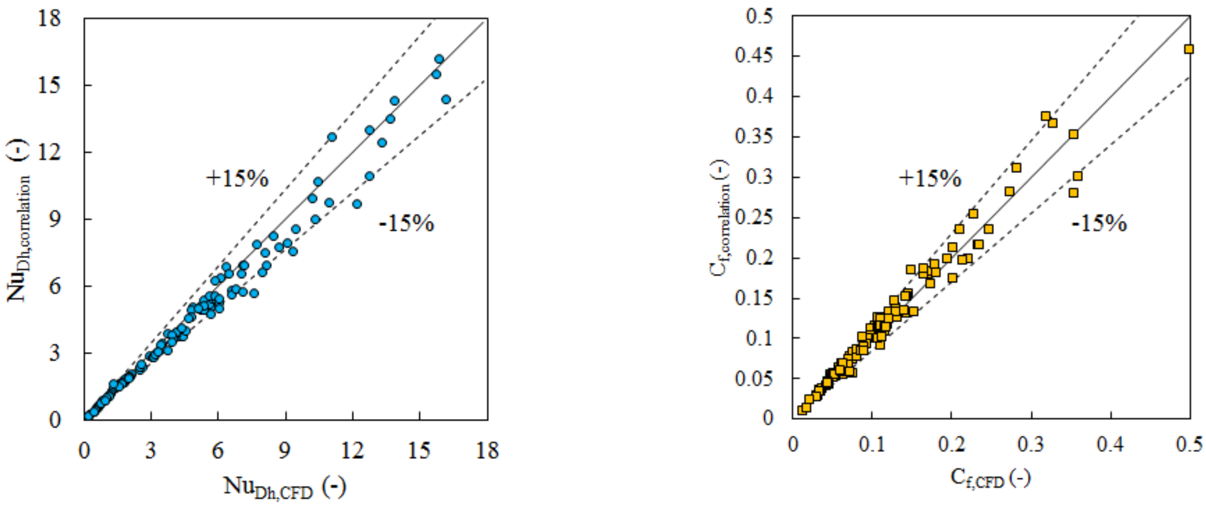


Figure 6: Herringbone correlations verification against 120 random designs.

6. DISCUSSION

In this section, we present a comparison between the proposed correlations with existing ones for the design space investigated in this paper. The existing correlations not only largely deviate from the CFD simulations but they are not even consistent within themselves, thus proving they are not applicable for this design space. Figure 7 presents the results for this analysis.

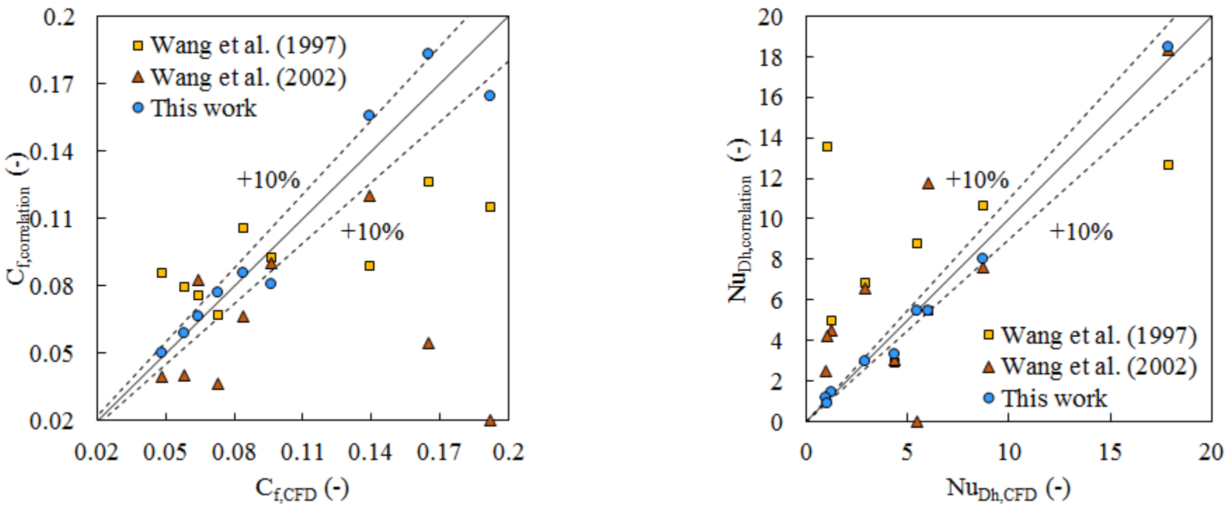


Figure 7: Herringbone correlations prediction comparison for sample designs from the source data.

7. CONCLUSIONS

This paper presented novel CFD-based correlations for smooth and Herringbone wavy fin and tube heat exchangers, in staggered arrangement. The main contribution is the design space investigated: Do = 2.0-5.0mm, Pl/Do and Pt/Do = 1.25-4.0, N = 2-20, Fp = 0.5-2.5mm, Pd = 0.088-0.84, δf = 0.05-0.1mm, u=0.5-7.0m/s. There are no correlations in the literature covering such design space. All correlations predict more than 93% of the source data within 15%. The verification against random samples resulted in similar fitness. Lastly, we demonstrated that the existing correlations are not applicable to the design space investigated in this paper, thus justifying the need for the new equations.

NOMENCLATURE

A_c	Minimum free flow area	m²	P_l	Tube longitudinal pitch	-
A_{fin}	Fin surface area	m²	P_t	Tube transverse pitch	-
A_{fr}	Frontal face area	m²	R	Grid refinement ratio	-
A_o	Surface area	m²	Re	Reynolds Number	-
C	Heat capacitance rate	W/K	T	Temperature	K
C_f	Friction factor	-	u	Velocity	m/s
c_p	Specific heat	J/kg.K	UA	Thermal conductance	W/K
d	Depth	mm	u_c	Maximum velocity	-
D_h	Surface hydraulic diameter	mm	X_f	Half wavelength	mm
Do	Tube outer diameter	-	ΔP	Pressure drop	Pa
e	Absolute relative difference	-			
F_p	Fin pitch	-			
F_s	Grid factor of safety	-	*Greek Letters*		
GCI	Grid Convergence Index	-	δ_f	Fin thickness	mm
h	Heat transfer coefficient	W/m².K	ε	Effectiveness	-
k	Air conductivity	W/m.K	σ	Contraction ratio (u/u_{max})	-
N	Number of tube banks	-	η	Fin efficiency	-
NTU	Number of tranfer units	-	η_o	Fin effectiveness	-
Nu_{Dh}	Nusselt number	-	φ	Correlated function	-
p	Order of accuracy	-	μ	Dynamic viscosity	Pa.s
P_d	Wave amplitude	mm	ρ	Density	kg/m³

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APPENDIX

Table 4: Herringbone correlation: parameters power matrices.

$M_{NuDh,Herr,N=2-10} =$

0	0	0	0	0	0	0
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
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0	2	0	0	0	0	0
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1	0	0	0	0	0	2
0	1	0	0	0	0	2
0	0	1	0	0	0	2
0	0	0	0	1	0	2
0	0	0	0	0	0	3

$M_{NuDh,Herr,N=11-20} =$

0	0	0	0	0	0	0
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
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0	1	0	1	0	0	1
0	0	1	1	0	0	1
0	0	0	2	0	0	1
1	0	0	0	1	0	1
1	0	0	0	0	1	1
0	1	0	0	0	1	1
0	0	1	0	0	1	1
1	0	0	0	0	0	2
0	1	0	0	0	0	2
0	0	1	0	0	0	2
0	0	0	1	0	0	2
0	0	0	0	0	0	3

$M_{Cf,Herr,N=2-10} =$

0	0	0	0	0	0	0
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1
2	0	0	0	0	0	0
1	1	0	0	0	0	0
0	2	0	0	0	0	0
1	0	1	0	0	0	0
0	1	1	0	0	0	0
0	0	2	0	0	0	0
1	0	0	1	0	0	0
0	1	0	1	0	0	0
0	0	1	1	0	0	0
0	0	0	2	0	0	0
1	0	0	0	1	0	0
0	1	0	0	1	0	0
0	0	1	0	1	0	0
0	0	0	1	1	0	0
0	0	0	0	2	0	0
1	0	0	0	0	1	0
0	1	0	0	0	1	0
0	0	1	0	0	1	0
0	0	0	1	0	1	0
0	0	0	0	1	1	0
0	0	0	0	0	2	0
1	0	0	0	0	0	1
0	1	0	0	0	0	1
0	0	1	0	0	0	1
0	0	0	1	0	0	1
0	0	0	0	1	0	1
0	0	0	0	0	1	1
0	0	0	0	0	0	2
3	0	0	0	0	0	0
2	1	0	0	0	0	0
2	0	1	0	0	0	0
0	2	1	0	0	0	0
1	0	2	0	0	0	0
2	0	0	1	0	0	0
1	1	0	1	0	0	0
1	0	1	1	0	0	0
0	1	1	1	0	0	0
0	0	2	1	0	0	0
1	0	0	2	0	0	0
0	1	0	2	0	0	0
0	0	1	2	0	0	0
1	1	0	0	1	0	0
0	2	0	0	1	0	0
1	0	1	0	1	0	0
0	1	1	0	1	0	0
0	0	2	0	1	0	0
0	1	0	1	1	0	0
1	0	0	0	2	0	0
0	0	1	0	2	0	0
0	0	0	1	2	0	0
0	2	0	0	0	1	0
0	1	1	0	0	1	0
1	0	0	1	0	1	0
0	1	0	1	0	1	0
0	0	1	1	0	1	0
1	0	0	0	1	1	0
1	0	0	0	0	2	0
0	1	0	0	0	2	0
0	0	1	0	0	2	0
2	0	0	0	0	0	1
1	1	0	0	0	0	1
0	2	0	0	0	0	1
1	0	1	0	0	0	1
0	1	1	0	0	0	1
0	0	2	0	0	0	1
0	1	0	1	0	0	1
0	0	1	1	0	0	1
0	0	0	2	0	0	1
0	0	0	1	1	0	1
1	0	0	0	0	1	1
0	1	0	0	0	1	1
0	0	0	1	0	1	1
1	0	0	0	0	0	2
0	0	1	0	0	0	2
0	0	0	0	0	0	3

$M_{Cf,Herr,N=11-20} =$

0	0	0	0	0	0	0
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1
2	0	0	0	0	0	0
1	1	0	0	0	0	0
0	2	0	0	0	0	0
1	0	1	0	0	0	0
0	1	1	0	0	0	0
0	0	2	0	0	0	0
1	0	0	1	0	0	0
0	1	0	1	0	0	0
0	0	1	1	0	0	0
0	0	0	2	0	0	0
1	0	0	0	1	0	0
0	1	0	0	1	0	0
0	0	1	0	1	0	0
0	0	0	1	1	0	0
0	0	0	0	2	0	0
1	0	0	0	0	1	0
0	1	0	0	0	1	0
0	0	1	0	0	1	0
0	0	0	1	0	1	0
0	0	0	0	1	1	0
0	0	0	0	0	2	0
1	0	0	0	0	0	1
0	1	0	0	0	0	1
0	0	1	0	0	0	1
0	0	0	1	0	0	1
0	0	0	0	1	0	1
0	0	0	0	0	1	1
0	0	0	0	0	0	2
2	1	0	0	0	0	0
1	2	0	0	0	0	0
2	0	1	0	0	0	0
1	1	1	0	0	0	0
0	2	1	0	0	0	0
1	0	2	0	0	0	0
2	0	0	1	0	0	0
1	1	0	1	0	0	0
0	2	0	1	0	0	0
1	0	1	1	0	0	0
1	0	0	2	0	0	0
0	1	0	2	0	0	0
2	0	0	0	1	0	0
1	0	1	0	1	0	0
0	0	2	0	1	0	0
0	0	1	0	2	0	0
1	1	0	0	0	1	0
0	1	1	0	0	1	0
1	0	0	1	0	1	0
0	1	0	0	1	1	0
0	0	1	0	1	1	0
0	0	0	1	1	1	0
0	1	0	0	0	2	0
0	0	0	0	0	3	0
2	0	0	0	0	0	1
1	1	0	0	0	0	1
0	1	0	1	0	0	1
0	0	1	1	0	0	1
0	0	0	2	0	0	1
0	0	1	0	1	0	1
0	0	0	0	2	0	1
1	0	0	0	0	0	2
0	1	0	0	0	0	2
0	0	1	0	0	0	2
0	0	0	1	0	0	2
0	0	0	0	0	1	2
0	0	0	0	0	0	3

Table 5: Herringbone correlation: coefficients arrays.

$c_{NuDh,Herr,N=2\text{-}10}$ =	$c_{NuDh,Herr,N=11\text{-}20}$ =	$c_{Cf,Herr,N=2\text{-}10}$ =	$c_{Cf,Herr,N=11\text{-}20}$ =
-4.3884	29.6938	17.8784	31.2430
3.8241	0.4421	0.0984	-3.6596
-5.5505	-0.8698	-4.1406	12.9874
6.8328	4.4086	-7.9936	-3.5077
-2.6098	1.6331	0.9576	-3.2856
0.0329	4.4418	2.4552	0.6230
-1.1233	-28.0007	-0.1146	-24.6064
1.7374	-1.1637	-5.0085	-6.4867
-0.6462	-0.4560	0.4685	1.0003
1.3482	1.9658	-0.4852	1.5960
0.0349	-0.6269	1.0845	-0.0451
-2.1053	-0.1432	-3.3757	-2.4313
2.7768	0.6684	2.1409	-0.5294
-6.2859	-6.4044	0.7065	1.2628
-0.0234	0.3081	0.0071	-0.6681
0.2687	-0.8489	-0.3017	0.4837
0.5023	-0.4941	0.4226	-0.1296
-0.6451	-0.3928	0.2527	0.3392
0.2754	1.0420	0.1028	0.6159
-0.5167	0.2411	-0.5853	0.5188
-0.8092	-0.5215	-2.7741	-1.1939
-0.3741	1.0004	0.0621	-0.6716
-0.1482	-0.2538	0.1284	0.3266
0.8253	0.5044	0.3281	0.6305
-1.3296	0.7808	-0.3773	-6.5030
0.3194	-0.3608	0.6310	-1.2588
0.1442	-1.1358	-0.1369	0.8830
-0.4880	9.0281	0.0885	0.8434
-0.1178	0.5064	-0.0921	10.5513
-0.9219	-0.0110	0.5127	1.2430
1.2285	0.2786	0.2217	-0.8330
-0.7660	-0.2996	0.4828	0.8205
0.2283	0.0725	0.0237	0.5328
-0.0099	0.2702	-0.0466	-0.1318
0.1554	0.2514	0.1258	0.6846
-0.2247	0.2106	0.6353	0.8623
0.0849	-0.3065	-0.1302	0.1633
-0.2555	0.2558	0.1589	-0.1833
-0.3418	-0.0661	-0.1541	-0.1804
0.2761	0.1316	-0.2403	-0.0977
-0.4108	-0.3520	0.6301	-0.2400
1.7931	-0.3499	-0.1232	0.5677
-0.0554	0.1945	0.1398	-0.2066
0.0616	1.7012	0.0434	0.3265
0.0608	-0.1004	0.1435	-0.1740
0.0762	0.0816	-0.2158	0.1582
0.1009	0.1131	0.1848	0.1835
-0.0712	-0.0637	-0.1618	-0.1225
-0.2624	0.1141	0.1107	0.0856
0.0965	0.0383	-0.1550	-0.3672
0.0831	-0.0478	0.1265	0.4759
-0.1091	-0.0449	-0.3860	-0.1495
-0.0279	-0.0742	0.1174	-0.2621
-0.0984	0.1513	0.6264	0.3994
0.1016	0.1741	-0.0665	0.2041
0.1421	-0.0782	-0.0796	-0.2314
-0.3254	-0.0746	-0.1753	-0.2792
0.2591	0.2177	-0.0224	0.2427
-0.3288	-0.5673	0.1808	0.9620
0.1320	0.8286	-0.1769	-1.4322
-0.1418	-0.5867	0.1315	-0.1285
0.0912	-0.1361	-0.1098	0.0561
-0.0458	0.2567	0.0918	-0.0546
-0.1132	-0.1054	0.0656	0.0814
-0.0965	-0.2954	-0.0820	0.0313
0.1953	-0.2674	0.1100	-0.0376
-0.1630	-1.1494	-0.1341	-0.0260
0.0889	0.0957	-0.1245	-0.0930
-0.1304	-0.2368	0.1035	0.0486
0.0992	0.2510	-0.1045	-0.0695
-0.1002	0.0344	0.0881	-0.0289
0.1873	-0.0405	-0.0973	-0.0468
-0.1797	0.1007	0.1147	-0.0389
0.1826	0.0180	-0.0356	
-0.0255	0.0466	0.0379	
0.0476	0.0824	0.0335	
0.0682	-0.1992	-0.0237	
0.0255	0.0993	0.0362	
0.0356	-0.0351	-0.0376	
-0.1093	0.0358	0.0439	
0.1012	-0.0684	-0.0501	
0.0357	0.0258	-0.0386	
0.0727	-0.0109	-0.0291	
-0.0722			
0.0415			
0.0173			
0.0153			

Table 6: Smooth correlation: parameters power matrices.

$M_{NuDh,Smooth,N=2-10} =$

0	0	0	0	0	0	0
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1
2	0	0	0	0	0	0
1	1	0	0	0	0	0
0	2	0	0	0	0	0
1	0	1	0	0	0	0
0	1	1	0	0	0	0
0	0	2	0	0	0	0
1	0	0	1	0	0	0
0	1	0	1	0	0	0
0	0	1	1	0	0	0
0	0	0	2	0	0	0
1	0	0	0	1	0	0
0	1	0	0	1	0	0
0	0	1	0	1	0	0
0	0	0	1	1	0	0
0	0	0	0	2	0	0
1	0	0	0	0	1	0
0	1	0	0	0	1	0
0	0	1	0	0	1	0
0	0	0	1	0	1	0
0	0	0	0	1	1	0
0	0	0	0	0	2	0
1	0	0	0	0	0	1
0	1	0	0	0	0	1
0	0	1	0	0	0	1
0	0	0	1	0	0	1
0	0	0	0	1	0	1
0	0	0	0	0	1	1
0	0	0	0	0	0	2
3	0	0	0	0	0	0
1	1	1	0	0	0	0
0	2	1	0	0	0	0
0	1	2	0	0	0	0
0	0	3	0	0	0	0
2	0	0	1	0	0	0
1	1	0	1	0	0	0
1	0	1	1	0	0	0
1	0	0	2	0	0	0
0	0	1	2	0	0	0
0	0	0	3	0	0	0
1	0	1	0	1	0	0
0	0	2	0	1	0	0
0	0	1	1	1	0	0
0	0	1	0	2	0	0
0	0	0	1	2	0	0
0	2	0	0	0	1	0
1	0	1	0	0	1	0
0	1	1	0	0	1	0
0	0	2	0	0	1	0
1	0	0	1	0	1	0
0	1	0	1	0	1	0
0	0	1	1	0	1	0
0	0	0	2	0	1	0
1	0	0	0	1	1	0
0	1	0	0	1	1	0
1	0	0	0	0	2	0
0	0	1	0	0	2	0
0	0	0	1	0	2	0
2	0	0	0	0	0	1
1	1	0	0	0	0	1
1	0	1	0	0	0	1
0	1	1	0	0	0	1
0	0	2	0	0	0	1
0	1	0	1	0	0	1
0	0	0	2	0	0	1
0	0	0	1	1	0	1
0	0	0	0	2	0	1
1	0	0	0	0	1	1
0	1	0	0	0	1	1
0	0	0	1	0	1	1
0	0	0	0	1	1	1
1	0	0	0	0	0	2
0	1	0	0	0	0	2
0	0	0	1	0	0	2

$M_{NuDh,Smooth,N=11-20} =$

0	0	0	0	0	0	0
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1
2	0	0	0	0	0	0
1	1	0	0	0	0	0
0	2	0	0	0	0	0
1	0	1	0	0	0	0
0	1	1	0	0	0	0
0	0	2	0	0	0	0
1	0	0	1	0	0	0
0	1	0	1	0	0	0
0	0	1	1	0	0	0
0	0	0	2	0	0	0
1	0	0	0	1	0	0
0	1	0	0	1	0	0
0	0	1	0	1	0	0
0	0	0	1	1	0	0
0	0	0	0	2	0	0
1	0	0	0	0	1	0
0	1	0	0	0	1	0
0	0	1	0	0	1	0
0	0	0	1	0	1	0
0	0	0	0	1	1	0
0	0	0	0	0	2	0
1	0	0	0	0	0	1
0	1	0	0	0	0	1
0	0	1	0	0	0	1
0	0	0	1	0	0	1
0	0	0	0	1	0	1
0	0	0	0	0	1	1
0	0	0	0	0	0	2
3	0	0	0	0	0	0
2	1	0	0	0	0	0
1	2	0	0	0	0	0
0	3	0	0	0	0	0
2	0	1	0	0	0	0
1	1	1	0	0	0	0
0	2	1	0	0	0	0
1	0	2	0	0	0	0
0	1	2	0	0	0	0
0	0	3	0	0	0	0
1	1	0	1	0	0	0
0	2	0	1	0	0	0
1	0	1	1	0	0	0
0	1	1	1	0	0	0
0	1	0	2	0	0	0
0	0	0	3	0	0	0
2	0	0	0	1	0	0
1	1	0	0	1	0	0
0	1	1	0	1	0	0
1	0	0	1	1	0	0
0	0	0	1	2	0	0
1	1	0	0	0	1	0
1	0	1	0	0	1	0
0	1	1	0	0	1	0
1	0	0	1	0	1	0
0	1	0	1	0	1	0
1	0	0	0	1	1	0
0	0	1	0	1	1	0
0	0	0	1	1	1	0
0	0	0	0	2	1	0
0	0	0	0	0	3	0
1	1	0	0	0	0	1
1	0	1	0	0	0	1
0	1	1	0	0	0	1
0	0	2	0	0	0	1
1	0	0	1	0	0	1
0	1	0	1	0	0	1
0	0	0	2	0	0	1
1	0	0	0	1	0	1
0	0	0	0	2	0	1
1	0	0	0	0	1	1
0	1	0	0	0	1	1
0	0	1	0	0	1	1
0	0	0	1	0	1	1
1	0	0	0	0	0	2
0	1	0	0	0	0	2
0	0	0	1	0	0	2

$M_{Cf,Smooth,N=2-10} =$

0	0	0	0	0	0	0
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1
2	0	0	0	0	0	0
1	1	0	0	0	0	0
0	2	0	0	0	0	0
1	0	1	0	0	0	0
0	1	1	0	0	0	0
0	0	2	0	0	0	0
1	0	0	1	0	0	0
0	1	0	1	0	0	0
0	0	1	1	0	0	0
0	0	0	2	0	0	0
1	0	0	0	1	0	0
0	1	0	0	1	0	0
0	0	1	0	1	0	0
0	0	0	1	1	0	0
0	0	0	0	2	0	0
1	0	0	0	0	1	0
0	1	0	0	0	1	0
0	0	1	0	0	1	0
0	0	0	1	0	1	0
1	0	0	0	0	0	1
0	1	0	0	0	0	1
0	0	1	0	0	0	1
0	0	0	1	0	0	1
0	0	0	0	1	0	1
0	0	0	0	0	1	1
0	0	0	0	0	0	2
3	0	0	0	0	0	0
2	1	0	0	0	0	0
2	0	1	0	0	0	0
1	1	1	0	0	0	0
1	0	2	0	0	0	0
2	0	0	1	0	0	0
1	1	0	1	0	0	0
1	0	1	1	0	0	0
0	1	1	1	0	0	0
0	0	2	1	0	0	0
1	0	0	2	0	0	0
0	1	0	2	0	0	0
0	0	0	3	0	0	0
1	1	0	0	1	0	0
0	2	0	0	1	0	0
1	0	1	0	1	0	0
0	0	2	0	1	0	0
0	0	1	1	1	0	0
0	0	1	0	2	0	0
0	0	0	1	2	0	0
2	0	0	0	0	1	0
0	2	0	0	0	1	0
0	0	2	0	0	1	0
1	0	0	1	0	1	0
0	1	0	1	0	1	0
0	0	1	1	0	1	0
2	0	0	0	0	0	1
1	1	0	0	0	0	1
0	1	1	0	0	0	1
1	0	0	1	0	0	1
0	1	0	1	0	0	1
0	0	1	1	0	0	1
0	0	0	1	0	1	1
1	0	0	0	0	0	2
0	0	1	0	0	0	2
0	0	0	1	0	0	2
0	0	0	0	1	0	2
0	0	0	0	0	0	3

$M_{Cf,Smooth,N=11-20} =$

0	0	0	0	0	0	0
1	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	1	0	0	0
0	0	0	0	1	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	1
2	0	0	0	0	0	0
1	1	0	0	0	0	0
0	2	0	0	0	0	0
1	0	1	0	0	0	0
0	1	1	0	0	0	0
0	0	2	0	0	0	0
1	0	0	1	0	0	0
0	1	0	1	0	0	0
0	0	1	1	0	0	0
0	0	0	2	0	0	0
1	0	0	0	1	0	0
0	1	0	0	1	0	0
0	0	1	0	1	0	0
0	0	0	1	1	0	0
0	0	0	0	2	0	0
1	0	0	0	0	1	0
0	1	0	0	0	1	0
0	0	1	0	0	1	0
0	0	0	1	0	1	0
0	0	0	0	1	1	0
0	0	0	0	0	2	0
1	0	0	0	0	0	1
0	1	0	0	0	0	1
0	0	1	0	0	0	1
0	0	0	1	0	0	1
0	0	0	0	0	1	1
0	0	0	0	0	0	2
3	0	0	0	0	0	0
2	1	0	0	0	0	0
1	2	0	0	0	0	0
1	1	1	0	0	0	0
0	2	1	0	0	0	0
1	0	2	0	0	0	0
2	0	0	1	0	0	0
1	1	0	1	0	0	0
0	2	0	1	0	0	0
1	0	1	1	0	0	0
0	1	1	1	0	0	0
1	0	0	2	0	0	0
0	1	0	2	0	0	0
0	0	1	2	0	0	0
0	0	0	3	0	0	0
1	0	1	0	1	0	0
0	0	2	0	1	0	0
0	1	0	1	1	0	0
0	0	1	0	2	0	0
0	1	0	1	0	1	0
1	0	0	0	1	1	0
0	0	1	0	1	1	0
0	0	0	0	2	1	0
0	1	0	0	0	2	0
2	0	0	0	0	0	1
1	1	0	0	0	0	1
1	0	1	0	0	0	1
0	1	1	0	0	0	1
0	0	2	0	0	0	1
1	0	0	1	0	0	1
0	1	0	1	0	0	1
0	0	1	1	0	0	1
1	0	0	0	0	1	1
0	0	0	1	0	1	1
1	0	0	0	0	0	2
0	0	0	0	0	0	3

Table 7: Smooth correlation: coefficients arrays.

$c_{NuDh,Smooth,N=2\text{-}10}$ =	$c_{NuDh,Smooth,N=11\text{-}20}$ =	$c_{Cf,Smooth,N=2\text{-}10}$ =	$c_{Cf,Smooth,N=11\text{-}20}$ =
-5.7057	32.2699	14.9492	11.7049
0.7473	2.0889	-0.9612	1.0108
-4.5714	-3.1603	-0.4285	7.8943
6.3303	4.1082	-5.0439	-0.3384
-3.2949	0.9142	-0.4762	-2.4832
-2.4175	3.2453	3.1646	4.6503
0.9464	-35.5205	-0.1368	1.2865
0.3337	0.7006	-4.6918	-4.6596
-0.6172	-1.0000	0.5159	-0.2212
1.9777	1.6918	-0.0760	1.8546
0.0537	-0.3625	0.1598	-0.3146
-0.9572	0.0047	-2.1545	-1.5804
2.7932	1.0831	0.7870	1.4594
-6.8360	-7.6816	0.3519	-0.0988
-0.0045	-0.2817	-0.8495	-1.2066
0.3399	-0.4810	0.4980	2.1100
0.5966	0.5453	0.6811	0.1105
-0.7793	-0.4627	0.7015	0.5710
0.0933	0.0978	0.6195	0.9489
0.0596	-0.1604	-0.5583	0.0665
-0.3317	-0.9331	-1.9039	-0.9244
-0.3773	-0.0080	-0.2504	-0.1326
-0.2579	0.3454	0.1632	0.7059
1.3956	-2.1642	0.0526	-1.7413
-1.4858	1.6837	-0.5721	-4.7211
0.4491	3.1080	-0.1079	-0.8095
0.0695	-1.1988	-0.3252	0.7196
0.4728	-1.8974	0.7386	-1.1130
-0.2903	9.8103	-0.1823	-0.6272
-0.3905	0.1866	0.6262	0.4534
1.3207	-0.0075	0.2840	-0.1177
-0.1277	-0.2406	-0.3062	0.0192
0.3739	-0.1284	0.0800	0.3436
0.2949	0.2087	0.6087	0.1479
0.0615	0.6082	-0.1902	0.6544
0.0571	-0.0295	0.1648	-0.2253
0.0945	-0.1252	-0.2062	0.4438
-0.3789	0.2862	-0.2885	-0.3248
-0.4326	-0.2511	0.4604	-0.2162
-0.3846	0.1345	-0.0767	-0.2127
1.9147	-0.0775	0.1900	0.4146
-0.0462	0.1997	0.1951	-0.2267
0.0663	-0.2700	0.0818	0.5053
0.0945	-0.3423	-0.2735	-0.2157
0.0895	0.2240	0.1020	0.3168
0.1016	1.8685	-0.1274	-0.1215
-0.1016	-0.1436	0.0458	0.1229
-0.1997	0.0875	-0.1859	-0.0968
-0.1353	0.2195	0.1968	0.0773
0.0979	-0.2477	-0.3822	0.0307
-0.0975	-0.0412	0.3896	-0.1800
-0.0315	-0.0428	0.0842	0.4379
0.1976	-0.1031	-0.1322	0.1251
-0.1907	0.1234	-0.0227	-0.0899
0.1413	0.2216	-0.0641	-0.1935
-0.3685	-0.1005	0.2633	-0.2338
0.1526	-0.0485	0.1523	-0.2705
-0.1252	-0.3657	0.1703	-0.2061
0.0773	0.6953	-0.0937	0.8248
-0.0741	-0.5997	0.1492	-0.1370
0.1074	-0.1672	-0.1325	0.0679
-0.1246	0.2013	0.0748	0.0801
0.1560	-0.3093	-0.0862	-0.1531
0.1376	0.2249	0.0726	0.1493
-0.1009	-0.1340	-0.1042	0.0842
0.0863	-0.2201	0.0351	-0.1070
-0.1619	-1.2405	0.0697	0.1313
0.1194	0.0950	-0.0492	0.1652
-0.2065	-0.1990	-0.0418	-0.0729
0.1284	0.2272	-0.0137	-0.0646
-0.0394	0.1003	0.0211	-0.0324
0.0525	0.0702	-0.0259	
0.0220	-0.0540		
0.0302	0.0248		
-0.1072	0.0569		
0.0476	0.0245		
0.0245	0.1427		
-0.0458	-0.2270		
0.0407	-0.1487		
-0.0945	0.0595		
-0.0088	-0.0197		
	0.0382		
	0.0141		